EFFECTIVE THERMAL CONDUCTIVITY OF COMMINUTED AGGREGATES

A. I. Tamarin

UDC 536.24:541.182

The steady-state heat flow through an aggregate of particles in a disordered arrangement is analyzed. A relation for calculating the effective thermal conductivity of such an aggregate is derived and compared with experimental data.

Many researchers have attempted to describe the laws which govern the heat transfer in aggregates. The model of a comminution where the thermal flux lines have a uniform spatial distribution was analyzed earlier. The bending of these lines in a real system was accounted for by assigning an appropriate geometrical shape to the particles: disks [1], cubes [3], parallelepipeds [2], spheres [5, 6], hollow bricks of square cross-section [7, 8], etc., with certain spatial distribution patterns. The relations for calculating the effective thermal conductivity of an aggregate are thoroughly summarized in [21, 20, 18, 8]. A comparison between experimental data and calculated results [20] shows wide discrepancies. An attempt will be made now to analyze the heat transfer in an aggregate by taking into account the random orientation of solid particles.

An aggregate of comminuted material usually contains particles of different sizes and shapes in a disordered arrangement. In a uniform steady-state thermal field the thermal flux lines in the aggregate follow diverse patterns. We will consider the heat transfer through an aggregate under moderate pressures and temperatures, when the heat exchange within the solid particles as well as in the gas phase can be described by the Fourier equation. We select a plane element of the aggregate whose thickness is on the order of a few particles. The mean heat flux through such a layer element is

$$\langle q \rangle = \langle \lambda \rangle \langle \operatorname{grad} T \rangle. \tag{1}$$

The brackets $\langle\ \rangle$ designate an average over the volume of the element. We next introduce a step function s whose value is unity within the gaseous interstices and zero within the solid particles. The steady-state heat flux through an aggregate of comminuted material may be expressed by the following relations:

in the solid mass

$$q(1-s) = \lambda_s \operatorname{grad} \theta, \tag{2}$$

in the gaseous mass

$$qs = \lambda \operatorname{grad} t.$$

After averaging over the volume of a layer element, we obtain for the gas

$$\langle qs \rangle = \lambda \langle \operatorname{grad} t \rangle,$$

and for the particles

$$\langle q(1-s) \rangle = \lambda_s \langle \operatorname{grad} \theta \rangle. \tag{3}$$

On the left-hand side of the equation we have the average value of the product of two functions. As is well known, this quantity can be replaced by the product of the average values of each function plus the product of their deviations from their respective average value

Institute of Heat and Mass Transfer, Academy of Sciences of the BSSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 5, pp. 823-827, May, 1970. Original article submitted June 2, 1969.

TABLE 1. Thermophysical Characteristics of Aggregates and Their Effective Thermal Conductivities

Number of test	$\frac{\lambda_4}{\lambda}$	$\langle\lambda\rangle$	λ_4	ε	r	$\frac{\lambda_2}{\langle\lambda\rangle}$	$\frac{\lambda_{20}}{\langle\lambda\rangle}$	Particle size, mm	Material	Reference
1	2	3	4	5	6	7	8	9	10	11
1	4	0,087	0,1	0,6	0,85	0,61	0,07	0,1	air–Teflon	[22]
2	5,04	0,097	0,126	0,51	0,864	0,96	0,34	0,2	air–polypropylene	[22]
3	7,3	0,095	0,19	0,4	0,621	2,56	1,16	0,05	air–Plexiglas balls	[8]
4	11	0,072	0,278	0,88	0,677	–0,6	–0,72	0,2	air–polyethylene	[22]
5	34,1	0,523	2,1	0,36	0,75	0,87	–0,48	0,292	air–ZrO_2 t = 600°C	[10]
6	39,3	0,152	1,51	0,71	0,67	–0,13	–0,35	0,2	CH_4–diatomic earth	[15]
7	41,5	0,13	1,04	0,61	0,64	0,02	–0,3	0,041	air–chamotte	[22]
8	43,6	0,18	1,09	0,4	0,705	0,79	0,15	3,8	air–glass balls	[4]
9	52,5	0,44	1,37	0,35	0,884	0,08	–0,66	0,78	air–quartz sand	[8]
10	68	0,234	1,09	0,4	0,865	–0,11	–0,44	0,17	CO_2–glass balls	[12]
11	66,6	0,291	2	0,36	0,75	0,64	0,0	0,292	air–ZrO_2 t = 100°C	[10]
12	92,8	0,099	1,51	0,71	0,774	–0,43	–0,58	0,2	CO_2–diatomic earth	[15]
13	136	2,34	26,2	0,37	0,803	0,18	–0,28	1,26	hydrogen–steel balls	[4]
14	151	0,695	9,3	0,42	0,807	0,04	–0,25	0,263	air–Al_2O_3 t = 600°C	[10]
15	156	1,0	28	0,38	0,72	1,56	0,55	0,65	oil–lead balls	[18]
16	184	2,18	35	0,35	0,765	0,46	–0,12	0,62	hydrogen–lead balls	[16]
17	700	0,655	26,5	0,38	0,85	–0,17	–0,49	3,18	CF_4–steel balls	[16]
18	945	0,42	30,2	0,42	0,82	–0,1	–0,44	0,263	air–Al_2O_3 t = 100°C	[10]
19	1180	0,493	34,3	0,417	0,87	–0,31	–0,56	6	air–lead balls	[17]
20	1260	0,418	34,3	0,642	0,845	–0,23	–0,52	1,6	air–lead balls	[14]
21	1460	0,52	38,4	0,38	0,87	–0,3	–0,49	3,2	air–steel balls	[16]
22	160	0,203	4	0,54	0,78	–0,13	–0,46	0,073	air–magnesite	[22]
23	2450	0,37	38,4	0,38	0,888	–0,38	–0,56	3,18	CO_2–steel balls	[16]
24	2770	0,206	74,3	0,66	0,776	–0,34	–0,62	0,079	air–iron	[22]
25	2780	0,195	69,5	0,59	0,784	–0,26	–0,49	0,044	air–ferrotitanium	[22]
26	420	0,204	10,5	0,54	0,762	–0,13	–0,42	0,287	air–corundum	[22]
27	15500	0,189	388	0,81	0,84	1,6	–0,8	0,026	air–copper	[22]

$$\langle qs\rangle = \langle q\rangle\langle s\rangle + \langle q's'\rangle,$$
$$\langle q(1-s)\rangle = \langle q\rangle\langle 1-s\rangle - \langle q's'\rangle. \quad (4)$$

Generally, the thermal conductivity of the particles is much greater than that of the gas, and the thermal flux lines crowd at the areas of contact between particles. As a result, even when a layer element is only several particles high, all flux lines will gradually pass through the gas interstices and the particles. There will not be a single flux line in the aggregate which passes only through the gas volume or only through the solid phase. It follows, therefore, that the mean temperature gradient across a layer element will be equal to the sum of the temperature gradients across the gas volume and across the solid volume:

$$\langle \operatorname{grad} T\rangle = \langle \operatorname{grad} t\rangle + \langle \operatorname{grad}\theta\rangle. \quad (5)$$

We solve Eqs. (3), (4), (5) simultaneously. After a few transformations, we finally arrive at

$$\frac{\lambda_4}{\langle\lambda\rangle\langle s\rangle} = \left(\frac{\lambda_4}{\lambda} - 1\right)\frac{\langle q's'\rangle}{\langle q\rangle\langle s\rangle} + \frac{\lambda_4}{\lambda} - \frac{\langle 1-s\rangle}{\langle s\rangle}. \quad (6)$$

The quantity $\langle s\rangle$ in Eq. (6) is the porosity and it expresses the fraction of total volume occupied by gas in the aggregate

$$\varepsilon = \langle s\rangle = \frac{1}{V}\int_V s\,dV. \quad (7)$$

On the right-hand side of Eq. (6) we have the quantity $\langle q's'\rangle/\langle q\rangle\langle s\rangle = r$, which is the correlation coefficient between the thermal flux and the solid-phase geometry in the aggregate. It expresses how strongly the thermal flux lines are coupled to the shape and the spatial arrangement of the particles.

Equation (6) can be rewritten as

$$\frac{\lambda_4}{\langle\lambda\rangle\varepsilon} - \frac{1-\varepsilon}{\varepsilon} = \frac{\lambda_4}{\lambda} + \left(\frac{\lambda_4}{\lambda} - 1\right) r. \tag{8}$$

It now relates the effective thermal conductivity $\langle\lambda\rangle$ of an aggregate to the thermophysical characteristics of the system. Since the thermal flux lines are strongly coupled to the solid-phase geometry, the value of the correlation coefficient should be near unity. The value of this coefficient can be determined from test data. Table 1 shows the experimental data which have been obtained by various researchers and which cover a wide range of thermophysical parameters describing the comminuted aggregate: the relative thermal conductivity ($4 < \lambda_4/\lambda < 15{,}500$), the porosity ($0.36 < \varepsilon < 0.88$), the particle size ($50\ \mu m < d < 6$ mm), and the layer temperature (20-600°C). At the same time, Table 1 gives values of the correlation coefficient calculated from experimental data and from Eq. (8). The calculated value is constant and near unity for all test conditions. The average value of the correlation coefficient is 0.785 with a variance (standard deviation) 8.8%. Considering all this, Eq. (8) may be written as

$$\langle\lambda\rangle = \frac{\lambda_4}{\varepsilon} \frac{1}{\frac{\lambda_4}{\lambda} + \frac{1-\varepsilon}{\varepsilon}\left(\frac{\lambda_4}{\lambda} - 1\right) 0.785}. \tag{9}$$

With this relation one can calculate the effective thermal conductivity of an aggregate over a wide range of variations in the thermophysical parameters of the fluid and of the solid component $\lambda_4/\lambda > 4$, $0.35 < \varepsilon < 0.9$. In order to explain the difference between experimental data and the calculation formula, we find the differential effective conductivity from Eq. (8) and, after a few transformations, the error in determining the effective thermal conductivity of an aggregate

$$\delta_{\langle\lambda\rangle} = \frac{r}{\frac{k}{k-1} + \frac{1-\varepsilon}{\varepsilon k} - r} \delta_r, \tag{10}$$

where δ_r is the relative error in calculating the correlation coefficient (the variance). The thermal conductivity of the solid phase is usually high and the ratio $k = \lambda_4/\lambda > 10$. The porosity of a layer varies between 0.35 and 0.9 and, therefore, $(1-\varepsilon)/\varepsilon$ is always smaller than 2. It follows that for $k \to \infty$

$$\delta_{\langle\lambda\rangle} \leqslant \frac{r}{1-r} \delta_r = 32\%.$$

Thus, the mean-square error in calculating the effective thermal conductivity of an aggregate by Eq. (9) does not exceed 32%.

We will now compare the experimental data with the calculated values found by other researchers. The calculation formulas suggested by a numer of authors are compared in [20], where it is shown that the formulas by Dul'nev [20] and Bogomolov [5] yield the closest agreement between theoretical and experimental values. These formulas together with the experimental data at hand were checked here. The calculated results are given in Table 1, where the subscripts 20 and 7 refer to values of effective thermal conductivity obtained by the formulas of [20] and [7] respectively.

The Dul'nev formula does not yield many points which have to be rejected, although the calculated values come out lower than the experimental ones over the entire range of λ_4/λ values. The standard deviation of experimental data from calculated ones is 51%.

The Bogomolov formula applies fairly generally to experimental data for aggregates of mineral material. Large deviations are observed for particles of irregular shape at high values of λ_4/λ (No. 27 in Table 1) and at low values of λ_4/λ (Nos. 1, 2, 3) and for a lead–oil system (No. 15). Consequently, the standard deviation between test and calculated data is relatively large here and equal to 77%.

Thus, it may be asserted that Eq. (9) yields a better approximation of experimental data. It is recommended for calculating the effective thermal conductivity of aggregates over a wide range of variations in the thermophysical parameters of the system: $0.35 < \varepsilon < 0.9$, $\lambda_4/\lambda > 4$.

It is to be noted here that Eq. (9) remains valid at various temperatures and pressures in the system, as long as the heat transfer through the contact points is negligibly small while the heat transfer through the gaseous interstices and the material is effected by conduction, i.e., as long as the Knudsen number is sufficiently small $Kn < 10^{-3}$.

NOTATION

$\langle \lambda \rangle$ is the effective thermal conductivity of an aggregate;
λ, λ_4 are the thermal conductivities of the gas and the particles respectively;
t, θ are the temperatures of the gas and the particles respectively;
T is the temperature of the aggregate;
q is the thermal flux
s is the "signum" function of the gas;
ε is the porosity of the aggregate;
$\langle\ \rangle$ is the averaging over the volume of an element.

LITERATURE CITED

1. O. Krischer and H. Rohnhalter, ZSVDF Forschungsheft, p. 402 (1940).
2. R. S. Bernshtein, Research on the Combustion of Natural Fuels [in Russian], Izd. GEI, Moscow (1948).
3. H. W. Russel, Jour. Amer. Ceramic Soc., 18, No. 1 (1935).
4. T. E. Schumann and V. Voss, Fuel in Sci. and Pract., 13, No. 6 (1934).
5. V. Z. Bogomolov, Trudy Fiz.-Agronom. Inst., No. 3 (1941).
6. D. Kunii and I. M. Smith, A. I. Ch. E. Jour., 6, No. 171 (1960).
7. G. N. Dul'nev, Inzh. Fiz. Zh., 9, No. 3 (1965).
8. L. V. Vasil'ev and Yu. E. Fraiman, Thermophysical Properties of Poor Heat Conductors [in Russian], Izd. Nauka i Tekhnika, Minsk (1967).
9. M. M. Fulk, Progress in Cryogenics, London (1959), Chap. 1.
10. H. W. Godbee and W. T. Ziegler, Jour. Appl. Phys., 37, No. 1 (1966).
11. W. G. Kannuluik and L. H. Martin, Proc. Roy. Soc., A141, 144 (1933).
12. L. A. Prins, I. Schenk, and J. G. L. Schran, Physica, 16, 397 (1950).
13. R. H. Kropschot, Cryogenic Technology, New York–London (1963), p. 243.
14. A. L. Waddams, Jour. Soc. Chem. Ind., 63, 337-340 (1944).
15. G. Hengst, Thermal Conductivity of Comminuted Thermal Insulation Materials at High Gas Pressure [in German], Munich (1934).
16. G. Kling, General Heat Engineering [in German], (1952), Chap. 3, pp. 167-174.
17. H. Verschot and G. C. A. Schuit, Appl. Sci. Res., A2 (1950).
18. A. Misnar, Thermal Conductivity of Solids, Liquids, Gases, and Their Mixtures [Russian translation], Izd. Mir, Moscow (1968).
19. V. I. Odelevskii, Zh. Tekh. Fiz., 21, No. 6 (1951).
20. G. E. Dul'nev and Z. V. Sigalova, Inzh. Fiz. Zh., 13, No. 5 (1967).
21. A. F. Chudnovskii, Thermophysical Characteristics of Comminuted Materials [in Russian], Izd. GIFML (1962).
22. S. S. Zabrodskii, G. I. Kovenskii, and A. I. Tamarin, in: Research on Transfer Processes in Apparatus with Comminuted Systems [in Russian], Izd. Nauka i Tekhnika, Minsk (1969).